E&CE316, Useful Formulae in Chapters 1-4[1]

1. The number of permutations of r objects from n distinct objects is

$$n(n-1)\cdots(n-r+1)=\frac{n!}{(n-r)!},\quad r\le n$$

2. The number of combinations of selecting r objects out of n is given by $\binom{n}{r}=\frac{n!}{(n-r)!r!}$
3. The number of ways to distribute n distinct balls into r different urns is r^n.
4. The number of ways to distribute n identical balls into r different urns without empty urns is $\binom{n-1}{r-1}$. If empty urns are allowed, then this number is $\binom{n+r-1}{r-1}$.
5. Probability of union of events:

$$P(E\cup F)=P(E)+P(F)-P(EF)$$
$$P(E\cup F\cup\cup G)=P(E)+P(F)+P(G)-P(EF)-P(EG)-P(FG)+P(EFG)$$

6. If $P(F)>0$, then $P(E|F)$, the conditional probability of event E given that F has occurred, is

$$P(E|F)=\frac{P(EF)}{P(F)}$$

7. Total probability:

$$P(E)=\sum_{i=1}^{n}P(E|F_i)P(F_i)$$

where the events $F_j, j=1,2,\cdots,n$, are a partition of the sample space.
8. Bayes' formula,

$$P(F_j|E)=\frac{P(EF_j)}{P(E)}=\frac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n}P(E|F_i)P(F_i)}$$

where the events $F_j, j=1,2,\cdots,n$, are a partition of the sample space.
9. Two events E and F are independent if $P(EF)=P(E)P(F)$.

10. $\sum_{i=1}^{n}=\frac{n(n+1)}{2}$, $e^x=\sum_{i=0}^{\infty}\frac{x^i}{i!}$, $\sum_{n=0}^{\infty}q^n=1/(1-q), 0<q<1$.

11.

Random variable X	Parameter	the p.m.f. $p(k)$	Expectation $E[X]$	Variance $Var(X)$
Binomial	(n,p)	$\binom{n}{k}p^k(1-p)^{n-k}$, $k=0,1,\cdots,n$	np	$np(1-p)$
Poisson	λ	$e^{-\lambda}\frac{\lambda^k}{k!}, k=0,1,,2,\cdots,$	λ	λ
Geometric	p	$(1-p)^{k-1}p, k=0,1,\cdots$	$1/p$	$(1-p)/p^2$

[1] This sheet will be attached with the mid-term exam.